Journal of Nonlinear Analysis and Optimization
Vol. 15, Issue. 4, No.1: 2024
ISSN: 1906-9685

ORTHOGONALITY OF GENERALIZED REVERSE (σ, τ) DERIVATIONS IN SEMIPRIME Γ– RINGS

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ABSTRACT:
Suppose that M is a semiprime Γ-ring. σ, τ are automorphisms of M. An additive mapping M is termed as a reverse (σ, τ)-derivation if it satisfies $d(u\alpha v) = d(v)\alpha\sigma(u) + \tau(v)\alpha\, d(u)$, for all $u, v \in M$ and $\alpha \epsilon \Gamma$. Moreover, an additive mapping $D : M \to M$ is termed a generalized reverse (σ, τ)- derivation if there exists a reverse (σ, τ)-derivation $d : M \to M$ such that $D(u\alpha v) = D(v)\alpha\sigma(u) + \tau(v)\alpha d(u)$, for all $u, v \in M$ and $\alpha \epsilon \Gamma$. This paper aims to extend the findings regarding orthogonal (σ,τ) derivations and orthogonal generalized (σ,τ)-derivations in semiprime Γ-rings to include the study of orthogonal reverse (σ,τ)-derivations and orthogonal generalized reverse (σ,τ)-derivations.

KEYWORDS: Semiprime Γ-ring, Reverse (σ, τ)-Derivation, Generalized Reverse (σ, τ)-Derivation, Orthogonal Reverse (σ, τ)-Derivation, Orthogonal Generalized Reverse (σ, τ)-Derivation.

1.INTRODUCTION:

Nobusawa [15] initially introduced the concept of a Γ−ring, where Γ denotes an additive abelian group. Barnes extended the results of Nobusawa on Γ−ring in his work [16]. M.Bresar and J.Vukman [11] have introduced the concept of orthogonal derivations and extended a theorem of Posner [6]. M.Soyturk [13] extended these results to Γ−rings. N.Argac et al. [14] introduced the notion of orthogonality for a pair of generalized derivations in semiprime rings and provided several necessary and sufficient conditions for the orthogonality of such pairs. Samman and Alyamani [12] studied the orthogonal conditions for reverse derivations in semi prime rings. Ashraf and Jamal [9] introduced the concept of orthogonality for two derivations in Γ-rings and established various necessary and sufficient conditions for the orthogonality of two derivations. Later they introduced orthogonal generalized derivation in Γ-rings in [10] and obtained results pertaining to orthogonal generalized derivations. K.K. Dey et al. [8] explored the conditions for the orthogonality of reverse derivations in semiprime Γ-rings. A.Shakir and M.S.Khan studied orthogonal (σ, τ)-derivations in semiprime Γ-rings. H.Yazarli and G.Oznur [7] studied orthogonal generalized (σ,τ)-derivations of semiprime Γ-rings. C. Jaya Subba Reddy and B. Ramoorthy Reddy [2,3,4,5] studied orthogonal symmetric biderivations and orthogonal generalized bi-(σ,τ) derivations in semiprime rings. In this paper, we expand the results of orthogonality of (σ,τ)derivations and generalized (σ,τ) derivations to reverse (σ,τ) derivations and generalized reverse (σ,τ) derivations in semiprime Γ – rings.

2.PRELIMINARIES:

Let M, Γ be additive abelian groups .

Suppose a mapping $M \times \Gamma \times M \rightarrow M : (u,\alpha,v) \rightarrow u\alpha v$ fulfils the conditions

1. $(u\alpha v)\beta w = u\alpha(v\beta w)$
2. $u(\alpha+\beta)v = u\alpha v + u\beta v,$
3. $(u+v)\alpha w = u\alpha w + v\alpha w,$
4. $u\alpha(v+w) = u\alpha v + u\alpha w$, for each $, v, w \in M, \alpha, \beta \in \Gamma$, then M is called a Γ ring.

M is referred to be semiprime if $u\alpha M\alpha u = 0$ suggests $u = 0$, for any $u \in M$. M is be 2-torsion free if 2u = 0 suggests u = 0 for any $u \in M$. An additive mapping $d: M \rightarrow$ M is said to be a reverse (σ,τ)- derivation of M if $d(u\alpha v) = d(v)\alpha\sigma(u) + \tau(v)\alpha d(u)$, for all $u, v \in M, \alpha \in \Gamma$. An additive mapping D: $M \rightarrow$ M is defined as a generalized reverse (σ,τ)- derivation if there exists a reverse (σ,τ)- derivation $d: M \rightarrow$ M such that $D(u\alpha v) = D(v)\alpha\sigma(u) + \tau(v)\alpha d(u)$, for all $u, v \in M, \alpha \in \Gamma$. If d_1, d_2 are two reverse (σ,τ)- derivations of M such that $d_1(u)\Gamma M\Gamma d_2(v) = d_2(v)\Gamma M\Gamma d_1(u) = 0$, for all $u, v \in M$ then d_1, d_2 are said to be orthogonal. If D_1, D_2 are two generalized reverse (σ,τ)- derivations of M such that $D_1(u)\Gamma M\Gamma D_2(v) = D_2(v)\Gamma M\Gamma D_1(u) = 0$, for all $u, v \in M$, then D_1, D_2 are said to be orthogonal.

Throughout the paper, M is assumed to be a semiprime $\boldsymbol{\Gamma}$-ring which is 2-torsion free and σ, τ are automorphisms of M. Also we assume that $d_1\tau = \tau d_1$; $d_2\tau = \tau d_2$; $d_1\sigma = \sigma d_1$; $d_2\sigma = \sigma d_2$. We denote a generalized reverse (σ,τ) derivations D_1, D_2 associated with the reverse (σ,τ)derivations d_1, d_2 such that $D_1\tau = \tau D_1$; $D_2\tau = \tau D_2$; $D_1\sigma = \sigma D_1$; $D_2\sigma = \sigma D_2$.

LEMMA 1 [[13], Lemma 3.4.1]: Suppose M is a semiprime Γ- ring with the condition 2a=0 for all $a \in M$ implies $a = 0$ and $a, b \in M$. Then the conditions listed below are mutually equivalent.

(i) $a\Gamma u\Gamma b = 0$, for all $u \in M$

(ii) $b\Gamma u\Gamma a = 0$, for all $u \in M$

(iii) $a\alpha u\beta b + b\alpha u\beta a = 0$, for all $u \in M$ and $\alpha, \beta \in \Gamma$.

If any of these conditions is attained, then $a\Gamma b = 0 = b\Gamma a$.

LEMMA 2 [**[13], Lemma 3.4.2**]**:** Let M be a semiprime Γ- ring and d_1 and d_2 be two additive mappings of M into itself satisfying $d_1(u)\Gamma M\Gamma d_2(u) = 0$ for all $u \in M$. Then $d_1(u)\Gamma M\Gamma d_2(v) = 0$, for all $u, v \in M$.

LEMMA 3: Suppose M is a semiprime Γ-ring which is 2-torsion free and d_1 and d_2 are reverse (σ,τ)- derivations of M. Then the following statements are biconditional.
1. d_1 and d_2 are orthogonal .
2. $d_1(u)\Gamma d_2(v)+d_2(u)\Gamma d_1(v)$= 0, for all $u,v\in M$.

Proof: we have to prove d_1 and d_2 are orthogonal ⇔ $d_1(u)\Gamma d_2(v)+\ d_2(u)\Gamma d_1(v)$ = 0
Suppose that d_1 and d_2 are orthogonal, then
$d_1(u)\Gamma M\Gamma d_2(v)=0=d_2(u)\Gamma M\Gamma d_1(v)$, for all $u,v\in M$
By Lemma 1, we can have $d_1(u)\Gamma d_2(v)=d_2(u)\Gamma d_1(v)=0$ and so
$d_1(u)\Gamma d_2(v)+d_2(u)\Gamma d_1(v)=0$
Conversely, suppose that $d_1(u)\Gamma\ d_2(v)+d_2(u)\Gamma\ d_1(v)$ = 0, for all $u,v\in M$
We can take $d_1(u)\beta d_2(v)+\ d_2(u)\gamma\ d_1(v)$ = 0, for all $u,v\in M$ and $\beta,\gamma\in\boldsymbol{\Gamma}$
(2.1)
Replacing v by $u\alpha v$ in (2.1), we attain
$d_1(u)\beta d_2(u\alpha v)+d_2(u)\gamma\ d_1(u\alpha v)$ = 0
$(d_1(u)\beta\ d_2(v)+d_2(u)\gamma\ d_1(v))\alpha\sigma(u)$ + $d_1(u)\beta\ \tau(v)\alpha d_2(u)+\ d_2(u)\gamma\ \tau(v)\alpha d_1(u)$ = 0
Using (2.1), we attain
$d_1(u)\beta\ \tau(v)\alpha d_2(u)+\ d_2(u)\gamma\ \tau(v)\alpha d_1(u)$ = 0
(2.2)
By substituting $\gamma=\ \gamma+\gamma'$ in equation (2.1), we arrive at
$d_2(u)\gamma^1 d_1(v)\alpha\sigma(u)$ = 0, for all $u,v\in M$ and $\alpha,\gamma^1\ \in\Gamma$
Hence, we can write $d_2(u)\gamma^1 d_1(v)\alpha\sigma(u)\alpha d_2(u)\gamma^1 d_1(v)\alpha\sigma(u)=0$
Given that σ is an automorphism of M and employing the semiprimeness of M , we obtain
$d_2(u)\Gamma d_1(v)=0$, for all $u\in M$
Again replacing β by $\beta+\beta'$ in equation (2.1), we arrive at
$d_1(u)\ \beta\ ^1 d_2(v)\alpha\sigma(u)$ = 0, for all $u,v\in M,\ \alpha,\beta^1\ \in\Gamma$ and hence we obtain
$d_1(u)\ \beta\ ^1 d_2(v)\alpha\sigma(u)\alpha d_1(u)\ \beta\ ^1 d_2(v)\alpha\sigma(u)=0$
Given that σ as an automorphism of M and using the semiprimeness of M, we attain
$d_1(u)\Gamma d_2(v)=0$, for all $u\ \in M$
Thus, we get $d_2(u)\Gamma d_1(v)=0=d_1(u)\Gamma d_2(v)$
Hence, we can conclude that $d_1\ and\ d_2$ are orthogonal.

LEMMA 4 : Suppose M is a semiprime Γ-ring free of 2-torsion and $(D_{1,}d_1)$ and $(D_{2,}d_2)$ are two orthogonal generalized reverse (σ,τ)- derivations of M, then the following relations are satisfied.
(i) $D_1(u)\Gamma D_2(v)=D_2(u)\Gamma D_1(v)=0$, hence $D_1(u)\Gamma D_2(v)+D_2(u)\Gamma D_1(v)=0$, for all $u,v\in M$
(ii) d_1 and D_2 are orthogonal and $d_1(u)\Gamma D_2(v)=D_{2,}(v)\Gamma d_1(u)=0$, for all $u,v\in M$
(iii) d_2 and D_1 are orthogonal and $d_2(u)\Gamma D_1(v)=D_{1,}(v)\Gamma d_2(u)=0$, for all $u,v\in M$
(iv) d_1 and d_2 are orthogonal
(v) $d_1D_2=\ D_2d_1=0$; $d_2D_1=\ D_1d_2=0$; $D_1D_2=\ D_2D_1=0$

Proof: (i) : Since $(D_{1,}d_1)$ and $(D_{2,}d_2)$ are orthogonal
By the definition of orthogonality of $D_{1,},D_2$, we can have
$D_1(u)\Gamma M\Gamma D_2(v)=0=D_2(v)\Gamma M\Gamma D_1(u),$ for all $u,v\in M$

By Lemma 1, we get $D_1(u)\Gamma D_2(v) = 0 = D_2(v)\Gamma D_1(u)$ and so
$D_1(u)\Gamma D_2(v) + D_2(v)\Gamma D_1(u) = 0$, for all $u, v \in M$
(ii) : Since (D_1, d_1) and (D_2, d_2) are orthogonal,
We can have $D_1(u)\Gamma M\Gamma D_2(v) = 0$ and so $D_1(u)\Gamma D_2(v) = 0$
Consider, $D_1(u)\alpha D_2(v) = 0$, for all $u, v \in M$ and $\alpha \epsilon \Gamma$ (2.3)
Replacing u by $u\beta r$ in (2.3), for all $u, r \in M$, $\beta \epsilon \Gamma$
$D_1(u\beta r)\alpha D_2(v) = 0$
$D_1(r)\beta\sigma(u)\alpha D_2(v) + \tau(r)\beta d_1(u)\alpha D_2(v) = 0$
Using the equation (2.3), we obtain
$\tau(r)\beta d_1(u)\alpha D_2(v) = 0$ and hence we can write (2.4)
$d_1(u)\alpha D_2(v)\beta\tau(r)\beta d_1(u)\alpha D_2(v) = 0$
Since τ is an automorphism of a semiprime $\boldsymbol{\Gamma}$- ring M, we get
$d_1(u)\alpha D_2(v) = 0$, for all $u, v \in M$, $\alpha \epsilon \Gamma$ (2.5)
Hence, d_1 *and* D_2 are orthogonal.
Replacing u by $r\beta u$, for u, $r \in M$ and $\beta \epsilon \Gamma$ in (2.5), we obtain
$d_1(r\beta u)\alpha D_2(v) = 0$, for $u, v, r \in M$, $\alpha, \beta \epsilon \Gamma$
$d_1(u)\beta\sigma(r)\alpha D_2(v) + \tau(u)\beta d_1(r)\alpha D_2(v) = 0$
Using the equation (2.5) in the above equation, we get
$d_1(u)\beta\sigma(r)\alpha D_2(v) = 0$, for $u, v, r \in M$, $\alpha, \beta \epsilon \Gamma$
Since σ is an automorphism, we get $d_1(u)\beta r\alpha D_2(v) = 0$
$d_1(u)\Gamma M\Gamma D_2(v) = 0$, for all $u, v \in M$
By Lemma 1, we obtain $d_1(u)\Gamma D_2(v) = 0 = D_2(v)\Gamma d_1(u)$, for all $u, v \in M$
(iii): By employing the similar procedure we adopted in the previous discussion, we can get
d_2 *and* D_1 are orthogonal and $d_2(u)\Gamma D_1(v) = 0 = D_1(v)\Gamma d_2(u)$, for all $u, v \in M$
(iv): We have D_1 and D_2 are orthogonal .
Hence, we can write $D_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$ (2.6)
Replacing u by $w\beta u$ and v by $x\gamma v$, for u, v, w, $x \in M$, $\alpha, \beta, \gamma \epsilon \Gamma$ in the equation (2.6), we get
$D_1(w\beta u)\alpha D_2(x\gamma v) = 0$, for all $u, v, w, x \in M$ and $\alpha, \beta, \gamma \epsilon \Gamma$
$D_1(u)\beta\sigma(w)\alpha D_2(v)\gamma\sigma(x) + \tau(u)\beta d_1(w)\alpha D_2(v)\gamma\sigma(x) + D_1(u)\beta\sigma(w)\alpha\tau(v)\gamma d_2(x) +$
$\tau(u)\beta d_1(w)\alpha\tau(v)\gamma d_2(x) = 0$
Using conditions (i), (ii), (iii) of Lemma 4, we get
$\tau(u)\beta d_1(w)\alpha\tau(v)\gamma d_2(x) = 0$, for all $u, v, w, x \in M$ and $\alpha, \beta, \gamma \epsilon \Gamma$
and hence $d_1(w)\alpha\tau(v)\gamma d_2(x)\beta\tau(u)\beta d_1(w)\alpha\tau(v)\gamma d_2(x) = 0$
Using the Semiprimeness of M and fact that τ is an automorphism,
we obtain $d_1(w)\alpha\tau(v)\gamma d_2(x) = 0$. This means $d_1(w)\Gamma M\Gamma d_2(x) = 0$
Hence, the proof is complete.
(v): We have (D_1, d_1) and (D_2, d_2) are orthogonal.
By (ii), we can write that d_1 and D_2 are orthogonal.
Hence, $d_1(u)\alpha w\beta D_2(v) = 0 = D_2(v)\alpha w\beta d_1(u)$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
Therefore, $D_2(D_2(v)\alpha w\beta d_1(u)) = 0$, for all $u, v, w \in M$ and $\alpha\ \beta \epsilon \Gamma$
$D_2(w\beta d_1(u))\alpha\sigma(D_2(v)) + \tau(w\beta d_1(u))\alpha d_2(D_2(v)) = 0$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
$D_2\big(d_1(u)\big)\beta\sigma(w)\alpha\sigma(D_2(v) + \tau\big(d_1(u)\big)\beta d_2(w)\alpha\sigma(D_2(v) + \tau(w\beta d_1(u))\alpha d_2(D_2(v)) = 0$
Since σ, τ are automorphisms and using $d_1\tau = \tau d_1$; $D_2\sigma = \sigma D_2$ we get

$D_2\big(d_1(u)\big)\beta w\alpha D_2(v) + d_1(u)\beta d_2(w)\alpha D_2(v) + w\beta d_1(u)\alpha d_2(D_2(v)) = 0$

Since d_1 , d_2 are orthogonal, we obtain $D_2\big(d_1(u)\big)\beta w\alpha D_2(v) = 0$ (2.7)

Replacing v by $d_1(u)$ in (2.7), we obtain

$D_2\big(d_1(u)\big)\beta w\alpha D_2\big(d_1(u)\big) = 0$

$D_2 d_1(u)\beta w\alpha D_2 d_1(u) = 0$

$D_2\big(d_1(u)\big)\Gamma M\Gamma D_2\big(d_1(u)\big) = 0$

By the semiprimeness of Γ-ring M, we can conclude that $D_2 d_1 = 0$

By following the similar procedure as we adopted in the earlier discussion, the results $d_1 D_2 = 0$; $d_2 D_1 = D_1 d_2 = 0$; $D_1 D_2 = D_2 D_1 = 0$ are evident.

Thus, we have $d_1 D_2 = 0 = D_2 d_1$; $D_1 d_2 = 0 = d_2 D_1$; $D_1 D_2 = D_2 D_1 = 0$

This completes the proof.

3.RESULTS:

THEOREM 1: Suppose M is a semiprime Γ-ring which is 2-torsion free and d_1 and d_2 are reverse (σ, τ)-derivations on M. Then the subsequent circumstances are equivalent:

1. d_1 and d_2 are orthogonal
2. $d_1 d_2 = 0$
3. $d_2 d_1$=0
4. $d_1 d_2 + d_2 d_1$=0
5. $d_1 d_2$ is a derivation.
6. $d_2 d_1$ is a derivation

Proof: Given that d_1 and d_2 are reverse (σ, τ)-derivations on M .

(1) ⟺ (2)

d_1 and d_2 are orthogonal ⟺ $d_1 d_2$ = 0

Suppose that d_1 and d_2 are orthogonal. Then we have

$d_1(\mathrm{u})\alpha v\beta d_2(w) = 0 = d_2(\mathrm{w})\alpha v\beta d_1(u)$, for all $u, v, w \in M$ and $\alpha, \beta \in \Gamma$

(3.1)

Hence, we can write $d_1(d_2(\mathrm{w})\alpha v\beta d_1(u))$=0

$d_1^2(u)\ \beta\sigma(d_2(\mathrm{w})\alpha v) + \tau\big(d_1(u)\big)\beta\ d_1(v)\alpha\sigma(d_2(\mathrm{w}) + \tau\big(d_1(u)\big)\beta\ \tau\ (v)\alpha d_1(d_2(\mathrm{w})) = 0$

Since σ, τ are automorphisms and using the equation (3.1), we get

$d_1(u)\beta v\alpha d_1(d_2(\mathrm{w})) = 0$

(3.2)

Replacing u by d_2(w) in equation (3.2), we get $d_1(d_2(\mathrm{w})\beta\ v\alpha d_1(d_2(\mathrm{w})) = 0$

$d_1 d_2(\mathrm{w})\beta\ v\alpha d_1 d_2(\mathrm{w}) = 0$, for all $v, w \in M$ and $\alpha\ \beta \in \Gamma$

By the semiprimeness of M, we can conclude that $d_1 d_2 = 0$

Conversely,

Suppose that $d_1 d_2 = 0$

Then $d_1 d_2(v\alpha u) = d_1\big(d_2(v\alpha u)\big)$

Since σ, τ are automorphisms of M, we get

$d_1 d_2(v\alpha u) = d_1(v)\alpha d_2(u) + v\alpha d_1 d_2(u) + d_1 d_2(v)\alpha\ u + d_2(v)\alpha d_1(u)$

Using the hypothesis that $d_1 d_2 = 0$, we get $d_1 d_2(v\alpha u) = d_1(v)\alpha d_2(u) + d_2(v)\alpha d_1(u)$

$d_1(v)\alpha d_2(u) + d_2(v)\alpha d_1(u) = 0$ and hence d_1 , d_2 are orthogonal.

(1) ⟺ (3)

By a similar argument, we can prove that d_1 and d_2 are orthogonal ⟺ $d_2\ d_1$= 0

(1) ⟺ (4)
d_1 and d_2 are orthogonal ⟺ $d_1d_2 + d_2d_1 = 0$
Suppose that d_1 and d_2 are orthogonal
By conditions (2),(3) of Theorem 1, we have already proved that $d_1d_2 = 0$ and $d_2d_1 = 0$ and hence $d_1d_2 + d_2d_1 = 0$
Conversely,
Suppose that $d_1d_2 + d_2d_1 = 0$, then we have to prove that d_1 and d_2 are orthogonal.
$(d_1d_2 + d_2d_1)(u\alpha v) = 0$
$d_1(\sigma(u))\alpha\sigma(d_2\,(v)) + \tau(\sigma(u))\alpha d_1d_2\,(v) + d_1d_2(u)\alpha\sigma(\tau(v)) + \tau(d_2(u)\alpha d_1(\tau(v)) +$
$d_2\big(\sigma(u)\big)\alpha\sigma(d_1(v)) + \tau(\sigma(u))\alpha\, d_2d_1(v) + d_2d_1(u)\alpha\sigma\big(\tau(v)\big) + \tau(d_1(u))\alpha d_2(\tau(v)) = 0$
$d_1(\sigma(u))\alpha\sigma(d_2\,(v) + \tau(\sigma(u))\alpha(d_1d_2 + d_2d_1)(v) + (d_1d_2 + d_2d_1)(u)\alpha\sigma(\tau(v)) +$
$\tau(d_2(u)\alpha d_1(\tau(v)) + d_2\big(\sigma(u)\big)\alpha\sigma(d_1(v)) + \tau(d_1(u))\alpha d_2(\tau(v)) = 0$
Since $d_1d_2 + d_2d_1 = 0$, we get
$d_1(\sigma(u))\alpha\sigma(d_2\,(v)) + \tau(d_2(u))\alpha d_1(\tau(v)) + d_2\big(\sigma(u)\big)\alpha\sigma(d_1(v)) + \tau(d_1(u))\alpha d_2(\tau(v)) = 0$
Since σ, τ are automorphisms of semiprime rings of M and using $d_1\sigma = \sigma d_1$; $d_2\sigma = \sigma d_2$; $d_1\tau = \tau d_1$; $d_2\tau = \tau d_2$, we obtain
$d_1(u)\alpha d_2\,(v) + d_2(u)\alpha d_1(v) + d_2(u)\alpha d_1(v) + d_1(u)\alpha d_2(v) = 0$
$2(d_1(u)\alpha d_2\,(v) + d_2(u)\alpha d_1(v)) = 0$
$d_1(u)\alpha d_2\,(v) + d_2(u)\alpha d_1(v) = 0$
Given that M is 2 torsion free , it follows that d_1, d_2 are orthogonal. (By Lemma 3)
(1) ⟺ (5)
d_1, d_2 are orthogonal ⟺ d_1d_2 is a derivation.
Suppose that d_1, d_2 are two orthogonal reverse (σ, τ) derivations
Then, $d_1(u)\alpha v\beta d_2\,(v) = 0 = d_2(v)\alpha v\beta d_1(u)$
$d_1d_2\,(u\alpha v) = d_1\big(d_2(u\alpha v)\big) = d_1(d_2(v)\alpha\sigma(u) + \tau(v)\alpha d_2(u))$
$= d_1(d_2(v)\alpha\sigma(u) + d_1(\tau(v)\alpha d_2(u))$
$= d_1\big(\sigma(u)\big)\alpha\sigma\big(d_2(v)\big) + \tau\big(\sigma(u)\big)\alpha d_1(d_2(v)) + d_1(d_2(u))\alpha\sigma(\tau(v) + \tau(d_2(u))\alpha d_1(\tau(v))$
Since σ, τ are automorphisms of semiprime $\boldsymbol{\Gamma}$-ring M and using $d_1\sigma = d_1\sigma$; $d_2\sigma = \sigma d_2$; $d_1\tau = \tau d_1$; $d_2\tau = \tau d_2$; we obtain
$d_1d_2\,(u\alpha v) = d_1(u)\alpha\, d_2(v) + u\alpha d_1d_2(v) + d_1d_2(u)\alpha v + d_2(u)\alpha d_1(v)$ (3.3)
$= d_1(u)\alpha\, d_2(v) + d_2(u)\alpha d_1(v) + d_1d_2(u)\alpha v + u\alpha d_1d_2(v)$
$= d_1d_2(u)\alpha v + u\alpha d_1d_2(v)$ (By orthogonality definition of $d_{1,}d_2$).
Therefore, $d_1d_2(u\alpha v) = d_1d_2(u)\alpha v + u\alpha d_1d_2(v)$. Hence, d_1d_2 is a derivation.
Conversely, suppose that d_1d_2 is a derivation
Then $d_1d_2\,(u\alpha v) = d_1d_2(u)\alpha v + u\alpha d_1d_2(v)$ (3.4)
Since d_1, d_2 are two reverse (σ, τ)-derivations, we can have
$d_1d_2\,(u\alpha v) = d_1(d_2(u\alpha v))$
$= d_1(u)\alpha\, d_2(v) + u\alpha d_1d_2(v)) + d_1d_2(u)\alpha v + d_2(u)\alpha d_1(v)$
Comparing the above equation with (3.3), we obtain
$d_1d_2(u)\alpha v + u\alpha d_1d_2(v) = d_1(u)\alpha\, d_2(v) + u\alpha d_1d_2(v) + d_1d_2(u)\alpha v + d_2(u)\alpha d_1(v)$
Hence, we get $d_1(u)\alpha\, d_2(v) + d_2(u)\alpha d_1(v) = 0$
Hence, $d_{1,}d_2$ are orthogonal.

(1) ⟺ (6)
d_1, d_2 are orthogonal ⟺ $d_2 d_1$ is a derivation.
Using the same procedure we adopted in the above proof, we can easily prove the result.

THEOREM 2:
Consider M as a semiprime Γ-ring which is 2 torsion free. There are two generalized reverse (σ,τ) derivations (D_1, d_1) and (D_2, d_2) of M that are orthogonal if and only if the following requirements are met.
(i) a) $D_1(u)\Gamma D_2(v) + D_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$
b) $d_1(u)\Gamma D_2(v) + d_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$
(ii) $D_1(u)\Gamma D_2(v) = d_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$
(iii) $D_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$ and $d_1 D_2 = d_1 d_2$

Proof: (i) : (D_1, d_1) and (D_2, d_2) are orthogonal ⟺ (i)
Suppose that (D_1, d_1) and (D_2, d_2) of M are orthogonal
By Using the conditions (i), (ii) and (iii) of Lemma 4, it is already proved that the two conditions
a) $D_1(u)\Gamma D_2(v) + D_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$
b) $d_1(u)\Gamma D_2(v) + d_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$ are satisfied.
Conversely
Suppose that the conditions
a) $D_1(u)\Gamma D_2(v) + D_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$
(3.5)
b) $d_1(u)\Gamma D_2(v) + d_2(u)\Gamma D_1(v) = 0$, for all $u, v \in M$ holds
(3.6)
Replacing u by $w\alpha u$ in (3.5), we get
$D_1(w\alpha u)\beta D_2(v) + D_2(w\alpha u)\beta D_1(v) = 0$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
$D_1(u)\alpha\sigma(w)\beta D_2(v) + \tau(u)\alpha(d_1(w)\beta D_2(v) + d_2(w)\beta D_1(v)) + D_2(u)\alpha\sigma(w)\beta D_1(v) = 0$
Using the equation (3.6), we get
$D_1(u)\alpha\sigma(w)\beta D_2(v) + D_2(u)\alpha\sigma(w)\beta D_1(v) = 0$
Since σ is an automorphism, we obtain
$D_1(u)\Gamma M\Gamma D_2(v) + D_2(u)\Gamma M\Gamma D_1(v) = 0$, for all $u, v \in M$
By Lemma 1, we can write $D_1(u)\Gamma D_2(v) = 0 = D_2(v)\Gamma D_1(u)$
Hence, we can conclude that D_1 and D_2 are orthogonal.
(ii) :
(D_1, d_1) and (D_2, d_2) are orthogonal⟺$D_1(u)\Gamma D_2(v) = d_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$.
Suppose that (D_1, d_1) and (D_2, d_2) are orthogonal. Hence by the conditions (i) and (ii) of Lemma 4, we can have
$D_1(u)\Gamma D_2(v) = 0$ and $d_1(u)\Gamma D_2(v) = 0$, for all u, $v \in M$.
(3.7)
Conversely,
Suppose that $D_1(u)\Gamma D_2(v) = 0$ and $d_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$.
Consider $D_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$
(3.8)

If we change u to $w\alpha u$ in (3.8), we obtain
$D_1(w\alpha u)\Gamma D_2(v) = 0$, for all $u, v, w \in M$ and $\alpha \epsilon \Gamma$
$D_1(u)\alpha\sigma(w)\beta D_2(v) + \tau(u)\alpha d_1(w)\beta D_2(v) = 0$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
Since σ is an automorphism and employing equation (3.7),we get
$D_1(u)\alpha w\beta D_2(v) = 0$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
Therefore, D_1, D_2 are orthogonal (By the definition of the orthogonality)
(iii):
(D_1, d_1) , $(D_2, d_2)$are orthogonal $\Leftrightarrow D_1(u)\Gamma D_2(v) = 0$ for all $u, v \in M$, $d_1D_2 = d_1d_2 = 0$
Suppose that (D_1, d_1) and $(D_2, d_2)$are orthogonal.
Then by the condition (i) and (v) of Lemma 4,
we can conclude that $D_1(u)\Gamma D_2(v) = 0$ and $d_1D_2 = 0$.
By the condition (iv) of Lemma 4, we conclude that d_1, d_2 are orthogonal
Hence, by Theorem 1, we can say that $d_1d_2 = 0$
Thus, we have proved $D_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M$, $d_1D_2 = d_1d_2 = 0$
Conversely,
Suppose that $D_1(u)\Gamma D_2(v) = 0$, for all $u, v \in M, d_1D_2 = d_1d_2 = 0$
(3.9)
Consider $d_1D_2 = 0$
Then $d_1D_2(\mathrm{u}\alpha\mathrm{v}) = d_1(D_2((\mathrm{u}\alpha\mathrm{v})) = d_1(D_2(\mathrm{v})\alpha\sigma(\mathrm{u}) + \tau(\mathrm{v})\alpha\mathrm{d}_2(\mathrm{u})) = 0$
$= d_1(\sigma(\mathrm{u}))\alpha\sigma(D_2(\mathrm{v})) + \tau(\sigma(\mathrm{u})\alpha\mathrm{d}_1(\mathrm{D}_2(\mathrm{v})) + d_1(\mathrm{d}_2(\mathrm{u}))\alpha\sigma(\tau(\mathrm{v})) + \tau(\mathrm{d}_2(\mathrm{u})\alpha d_1(\tau(\mathrm{v})) = 0$
Since σ, τ are automorphisms of semiprime rings of M and using $d_1\sigma = d_1\sigma$; $d_1\tau = \tau d_1$;$D_2\sigma = \sigma D_2$, we obtain
$d_1(\mathrm{u})\alpha D_2(\mathrm{v}) + \mathrm{u}\alpha\mathrm{d}_1\mathrm{D}_2(\mathrm{v}) + d_1\mathrm{d}_2(\mathrm{u})\alpha\mathrm{v} + \mathrm{d}_2(\mathrm{u})\alpha d_1(\mathrm{v}) = 0$
(3.10)
Using the equation (3.9), we get
$d_1(\mathrm{u})\alpha D_2(\mathrm{v}) + \mathrm{d}_2(\mathrm{u})\alpha d_1(\mathrm{v}) = 0$
(3.11)
By Theorem 1, if $d_1\mathrm{d}_2 = 0$, then d_1, d_2 are orthogonal and so equation (3.11) becomes
$d_1(\mathrm{u})\alpha D_2(\mathrm{v})=0$, for all $u, v \in M$ and $\alpha \epsilon \Gamma$
(3.12)
If we replace $u = w\beta u$ in (3.12) and using (3.12)
$d_1(w\beta u)\alpha D_2(v)=0$, for all $u, v, w \in M$ and $\alpha, \beta \epsilon \Gamma$
$d_1(\mathrm{u})\beta\sigma(\mathrm{w})\alpha D_2(\mathrm{v}) + \tau(\mathrm{u})\beta d_1(w)\alpha D_2(\mathrm{v}) = 0$
$d_1(\mathrm{u})\beta\sigma(\mathrm{w})\alpha D_2(\mathrm{v}) = 0$
Since σ is an automorphism and using Lemma 1, we can conclude that
$d_1(\mathrm{u})\Gamma\mathrm{M}\Gamma D_2(\mathrm{v}) = 0$ and so $d_1(\mathrm{u})\Gamma D_2(\mathrm{v}) = 0$
(3.13)
From (3.9) and (3.13) and using the condition (ii) of Theorem 2 we can conclude that
(D_1, d_1) and $(D_2, d_2)$are orthogonal.

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